

Generalised Inflation with a Gravitational Wave Background

V.N. Lukash¹, E.V. Mikheeva¹, V. Müller², and A.M. Malinovsky¹

¹*Astro Space Center of P.N. Lebedev Physical Institute*

Profsoyuznaya 84/32, 117810, Moscow, Russia

²*Astrophysikalisches Institute Potsdam, An der Sternwarte 16, Potsdam, D-14482 Germany*

Accepted. Received; in original form

ABSTRACT

We propose a Λ -inflation model which explains a significant part of the COBE signal by primordial cosmic gravitational waves. The primordial density perturbations fulfill both the constraints of large-scale microwave background and galaxy cluster normalization. The model is tested against the galaxy cluster power spectrum and the high-multipole angular CMB anisotropy.

1 INTRODUCTION

The observational reconstruction of the *cosmological density perturbation* (CDP) spectrum is a key problem of modern cosmology. It provided a dramatic challenge after detecting the primordial *cosmic microwave background* (CMB) anisotropy by DMR COBE (Smoot et al. 1992, Bennett et al. 1992) as the signal found at 10^0 , $\Delta T/T = 1.06 \times 10^{-5}$ (Bennett et al. 1996), appeared to be few times higher than the expected value of $\Delta T/T$ in the most simple and best developed cosmological model with standard *cold matter* (sCDM)^{*}. Indeed, the CDP spectrum of sCDM normalized by the biasing parameter $b^{-1} \equiv \sigma_8 = 0.6$ (White, Efstathiou, Frenk 1993, Eke, Coles, Frenk 1996, Viana, Liddle 1996) can reproduce only 30% of the COBE measured CMB anisotropy. If σ_8 is less than 0.6, this contradiction gets even stronger.

During recent years there were many proposals to improve sCDM[†] by adding hot dark matter, a Λ -term, or considering non-flat primordial CDP spectra. Below, we present another, presumably more natural way to solve the sCDM problem based on taking into account a possible contribution of *cosmic gravitational waves* (CGWs) into the large-scale CMB anisotropy. Further, we will also try to preserve the original near-scale-invariant CDP spectrum in the CDM universe. Thus, the problem is reduced to the construction of a simple inflation producing a near *Harrison-Zel'dovich* (HZ) spectrum of CDPs ($n_S \simeq 1$) and a large relative con-

tribution of CGWs into the $\Delta T/T$ at COBE scale (i.e. the ratio of the tensor to scalar mode contributions $T/S \sim 1$).

A basic physical reason for the production of tensor and scalar perturbations in the expanding Universe is the *parametric amplification effect* (Grishchuk 1974, Lukash 1980): the spontaneous creation of quantum physical fields in a non-stationary gravitational background. From the theoretical point of view a cosmology with negligible contribution of CGWs may be considered as a phenomenological model only, because any inflationary model predicts a non-zero CGW amplitude. Generally, the tensor mode is not discriminated in inflation (e.g. Starobinsky 1979, Rubakov, Sazhin, Veryaskin 1982). Different models with nearly scale-invariant spectra of CDPs predict different abundance of CGWs (Lucchin, Matarrese 1985, Linde 1994, Garsia-Bellido, Linde, Wands 1996, Lukash, Mikheeva 1996). However, in chaotic and power-law inflations it is usually small, $T/S \lesssim 1$, which corresponds to the spectrum slope $n \geq 0.8$. An always ‘red’ CDP spectrum ($n_S < 1$) originating in the power-law inflation model helps to satisfy the ‘double-normalization’ (i.e. to reconcile the σ_8 vs. $\Delta T/T$ problem).

So, the first candidate of a successful model fitting both COBE and galaxy cluster normalizations could be a power-law spectrum of CDPs with a slope $n_S \simeq 0.85$ predicted by power-law inflation (e.g. Lucchin, Matarrese 1985, there an exponential potential of the inflation field is used). One has a simple estimate for the fraction of CGWs,

$$\frac{T}{S} \simeq -6n_T, \quad (1)$$

where n_T is the slope of the CGW spectrum related trivially to n_S in the case of power-law inflation: $n_T = n_S - 1$.

Notice that large T/S and hence the desired double-normalization can be reached in this model only at the expense of a rejection from the HZ spectrum: the red power-law CDP spectrum helps on large scale, however it is getting undesirable on Mpc scale producing a too late galaxy

^{*} The matter density $\Omega_m = \Omega_b + \Omega_{cm} = 1$, there is no cosmological term, $h = 0.5$, the slope of the CDP spectrum $n_S = 1$, i.e. of Harrison-Zeldovich type, no contribution from cosmological gravitational waves.

[†] In the simplest terms, to remove the discrepancy between the CDP spectrum amplitudes at $8h^{-1}\text{Mpc}$ as determined by galaxy clusters, and at large scales, $\sim 1000h^{-1}\text{Mpc}$, by the $\Delta T/T$ inhomogeneity.

formation epoch (Gardner et al. 1997, and references cited therein).

Therefore, it is interesting to consider other models with high abundance of CGWs but at the same time with $n_S \simeq 1$ unchanged (with a preference of HZ or a slightly blue CDP spectrum on short scales to provide early formation of high-redshift quasars and early galaxy formation).

A simple model of such kind is Λ -inflation, an inflationary model with an effective metastable Λ -term (Lukash, Mikheeva 1996, 1997, 2000, Mikheeva 1997). This model produces both curvature and primordial gravitational wave perturbations which have a non-power-law spectrum, with a shallow minimum in the CDP spectrum, located at a scale k_{cr} (where the Λ -term and the scalar field have equal energy densities). Around this scale, the CDP spectrum is exactly of the scale invariant HZ form, and the ratio T/S is close to its maximum; it is of the order unity depending on the model parameters.

The estimate (1) remains true for Λ -inflation and, probably, keeps its universality for any type of inflationary dynamics (however, the relationship between n_T and n_S can vary). The cost to be paid for the possibility of having a high ratio T/S on scales where $n_S \simeq 1$ is the non-power-law CDP global spectrum: it is ‘red’ at $k < k_{cr}$ and ‘blue’ at $k > k_{cr}$. This smooth transition in the spectrum slope from red to blue makes T/S obviously dependent on how k_{COBE} is related to k_{cr} . One may benefit from the HZ local slope or a blue spectrum enhancement on Mpc scales (to initiate the early structure formation) by simply adjusting k_{cr} to a galaxy cluster scale.

2 Λ -INFLATION WITH SELF-INTERACTION

We summarize briefly the main properties of Λ -inflation and determine the basic model for our investigation. Let us consider a general potential of Λ -inflation:

$$V(\varphi) = V_0 + \sum_{\kappa=2}^{\kappa_{max}} \frac{\lambda_\kappa}{\kappa} \varphi^\kappa, \quad (2)$$

where φ is the inflaton scalar field, $V_0 > 0$ and λ_κ are constants, and $\kappa = 2, 3, 4, \dots$

In the case of a massive inflaton ($\kappa = \kappa_{max} = 2$, $\lambda_2 \equiv m^2 > 0$, this model is called Λm -inflation) T/S can be larger than unity only when the CDP spectrum slope in the ‘blue’ asymptote is very steep, $n_S^{blue} > 1.8$. To avoid such a strong spectral bend on short scales ($k > k_{cr}$), we choose here another simple version of Λ -inflation – the case with self-interaction: $\kappa = \kappa_{max} = 4$, $\lambda_4 \equiv \lambda > 0$; this model is called $\Lambda\lambda$ -inflation.

The scalar field φ drives an inflationary evolution if $\gamma \equiv -\dot{H}/H^2 < 1$, where[‡] $H = \sqrt{V/(3-\gamma)} \simeq \sqrt{V/3}$. This condition holds true for all values of φ (at worst, for $\varphi \sim \varphi_{cr}$, where $V_0 = \lambda\varphi_{cr}^4/4$ and γ reaches its maximum) if

[‡] We assume Planckian units $8\pi G = c = \hbar = 1$, a dot denotes the time derivative ($\equiv d/ad\eta$), and a and $H \equiv \dot{a}/a$ are scale and Hubble factors, respectively.

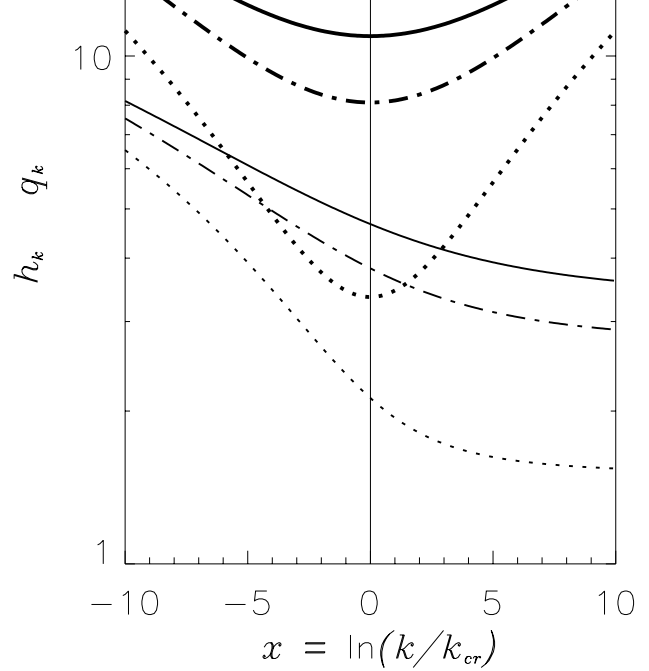


Figure 1. Spectra of curvature perturbations q_k (thick lines) and gravitational waves h_k (thin lines) around the typical scale k_{cr} for $c = 5, 9, 11$ as dotted, dash-dotted, and solid lines, respectively (arbitrary normalization).

$$c \equiv \frac{1}{4} \varphi_{cr}^2 = \frac{1}{2} \sqrt{\frac{V_0}{\lambda}} > 1, \quad (3)$$

which we imply hereafter.

The last inequality simultaneously ensures the validity of the slow-roll approximation ($|\ddot{\varphi}/\dot{\varphi}| \ll 3H$) as

$$\gamma \simeq \frac{2}{c} \left(\frac{y^3}{1+y^4} \right)^2, \quad \frac{\ddot{\varphi}}{3H\dot{\varphi}} \simeq -\frac{y^2(3+y^4)}{3c(1+y^4)^2}, \quad (4)$$

where $y \equiv \varphi/\varphi_{cr} = \varphi/(2\sqrt{c})$. Then we get the value of the inflaton field at horizon-crossing time ($k = aH \simeq -1/\eta$),

$$cy^2 = \sqrt{c^2 + x^2} - x,$$

and the gravitational perturbation spectra q_k and h_k generated in S and T modes, respectively (see the Appendix):

$$q_k = \frac{H}{2\pi\sqrt{2}\gamma} = \frac{\sqrt{2\lambda/3}}{\pi} (c^2 + x^2)^{3/4}, \quad (5)$$

$$h_k = \frac{H}{\pi\sqrt{2}} = \frac{2c\sqrt{\lambda/3}}{\pi} \left(1 + \frac{x}{\sqrt{c^2 + x^2}} \right)^{-1/2}, \quad (6)$$

where

$$\begin{aligned} x &= \ln \left[\frac{k}{k_{cr}y} \left(\frac{2}{1+y^4} \right)^{1/6} \right] \\ &= \ln \left[\frac{k}{k_{cr}} \left(1 + \left(\frac{x}{c} \right)^2 \right)^{1/4} \left(1 + \frac{x}{\sqrt{c^2 + x^2}} \right)^{2/3} \right] \\ &\simeq \ln(k/k_{cr}). \end{aligned}$$

Fig.1 shows the power spectra for $c = 5, 9, 11$.

The dimensionless power spectrum of CDPs is directly related to the fundamental q_k spectrum

$$\langle \delta^2 \rangle = \int_0^\infty \Delta_k^2 \frac{dk}{k}, \quad \Delta_k = 3.6 \times 10^6 \left(\frac{k}{h} \right)^2 q_k T(k), \quad (7)$$

where the wave number k is measured in h/Mpc , and $T(k)$ is the transfer function.

We can find also the local slopes of the fundamental power spectra (5), (6):

$$n_S - 1 \equiv 2 \frac{d \ln q_k}{d \ln k} = \frac{3x}{c^2 + x^2} \in \left[-\frac{3}{2c}, \frac{3}{2c} \right], \quad (8)$$

$$n_T \equiv 2 \frac{d \ln h_k}{d \ln k} = -2\gamma = -\frac{1}{\sqrt{c^2 + x^2}} \left(1 - \frac{x}{\sqrt{c^2 + x^2}} \right) \in \left[-\frac{3\sqrt{3}}{4c}, 0 \right]. \quad (9)$$

Obviously, the maximal deviations from the HZ spectrum take place in S-mode at $x = \pm c$, and in T-mode at $x = -c/\sqrt{3}$. At the latter point the ratio of spectra reaches its maximum,

$$\left(\frac{h_k}{q_k} \right)^2 = 4\gamma = 2|n_T| \leq \frac{3\sqrt{3}}{2c}. \quad (10)$$

3 CDM COSMOLOGY FROM Λ -INFLATION

Let us consider the CDP spectrum Δ_k (7) with CDM transfer function, normalized both at the large-scale $\Delta T/T|_{10^0}$ (including the contribution from h_k (6)) and the galaxy cluster abundance at $z = 0$ (σ_8), to find the family of the most realistic S-spectra q_k produced in $\Lambda\lambda$ -inflation.

In total, we have three parameters entering the q_k spectrum: λ (the overall amplitude), c (the measure of T/S) and k_{cr} (the scale where the CDP spectrum is locally HZ, $n_S = 1$ [§]). Constraining them by two observational tests, we are actually left with only one free parameter (say, k_{cr}) which may be restricted elsewhere by other observations (e.g. cluster power spectra, acoustic peaks, bulk velocities, etc.).

To demonstrate explicitly how the three parameters are mutually related, we first employ simple analytical estimates for the σ_8 and $\Delta T/T$ tests to derive the key equation relating c and k_{cr} , and then solve this equation numerically to obtain the range of interesting physical parameters.

Instead of taking the σ -integral numerically ($R = 8h^{-1}\text{Mpc}$),

$$\sigma_R^2 = \int_0^\infty \Delta_k^2 W^2(kR) \frac{dk}{k}, \quad W(z) = \frac{3}{z^3} (\sin z - z \cos z), \quad (11)$$

we may estimate the wavelength k_1 at which $\Delta_{k_1} = 1.6\sigma_8$ (to be equal unity for $\sigma_8 = 0.6$)[¶]. This will fix the spectrum amplitude on the cluster scale (see eq.(7)):

[§] $n_S = 1$ is also in the asymptotics $|x| \gg c$.

[¶] The integral is roughly estimated as $\sigma_8^2 \simeq \Delta_{k_1}^2 / \alpha$ assuming

$$q_{k_1} \simeq 4.5 \times 10^{-7} \frac{h^2 \sigma_8}{k_1^2 T(k_1)}. \quad (12)$$

On the other hand, the spectrum amplitude on large scale ($k_2 = k_{COBE} \simeq 10^{-3} h/\text{Mpc}$) can be taken from $\Delta T/T$ due to the Sachs-Wolfe-effect (Sachs, Wolfe 1967):

$$\left\langle \left(\frac{\Delta T}{T} \right)^2 \right\rangle_{10^0} = S + T \simeq 1.1 \times 10^{-10}, \quad S = 0.04 \langle q^2 \rangle_{10^0}. \quad (13)$$

The relationship between the power spectrum at COBE scale and the variance of the q potential averaged in 10^0 -angular-scale at the last scattering surface, involves an effective interval of the spectral wavelengths contributing to the latter:

$$\langle q^2 \rangle_{10^0}^{1/2} = f q_{k_2}, \quad f^2 \sim \ln \left(\frac{k_2}{k_{hor}} \right). \quad (14)$$

To estimate T/S, we will use the approximation formula (1) for $x_2 = x_{COBE}$ (cf. eqs.(9), (10)):

$$\begin{aligned} \frac{T}{S} &\simeq -6n_T = 12\gamma = 3 \left(\frac{h_{k_2}}{q_{k_2}} \right)^2 \\ &= \frac{6}{\sqrt{c^2 + x_2^2}} \left(1 - \frac{x_2}{\sqrt{c^2 + x_2^2}} \right). \end{aligned} \quad (15)$$

Evidently, both normalizations determine essentially the corresponding q_k amplitudes at the locations of cluster (k_1) and COBE (k_2) scales. Accordingly, the parameters k_1 and f can slightly vary while changing the local spectrum slopes at the respective wavelengths. However, when the deviation of the q_k slopes from HZ is small (e.g. $c > 5$, see eq.(8)) we can just identify the parameters k_1 and f for CDM models with their sCDM values:

$$k_1 \simeq 0.3h/\text{Mpc}, \quad f \simeq 1.26.$$

Finally, comparing the two normalization conditions with the theoretical spectrum q_k in eq.(5) we get the key equation for the relationship between c and k_{cr} ,

$$\left(\frac{q_{k_1}}{q_{k_2}} \right)^2 \simeq D \left(1 + \frac{T}{S} \right), \quad (16)$$

which looks like an algebraic equation for finding x_2 by given c :

$$\left(1 + \frac{d(d + 2x_2)}{c^2 + x_2^2} \right)^{3/2} \simeq D \left(1 + \frac{6}{\sqrt{c^2 + x_2^2}} - \frac{6x_2}{c^2 + x_2^2} \right). \quad (17)$$

Here $D \sim \sigma_8^2$, and

$$d = x_1 - x_2 \simeq \ln \left[300 \left(\frac{c^2 + x_2^2}{c^2 + x_1^2} \right)^{1/12} \left(\frac{\sqrt{c^2 + x_2^2} - x_2}{\sqrt{c^2 + x_1^2} - x_1} \right)^{2/3} \right].$$

The λ -parameter is then obtained as

$$\sqrt{\lambda} \simeq \frac{10^{-4} \sigma_8}{(c^2 + x_1^2)^{3/4}}.$$

Eq.(16) has a clear physical meaning: the ratio of the S-spectral power at cluster and COBE scales is proportional

that the integrand grows sharp with k and $\Delta_k^2 \sim k^\alpha$ near $k \sim k_1$. For sCDM, $\alpha \simeq 2.5$ at $k_1 \simeq 0.3h/\text{Mpc}$.

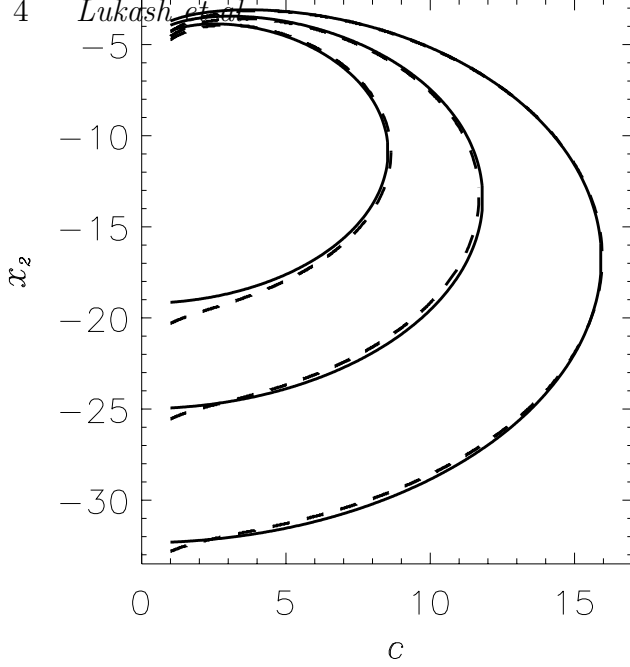


Figure 2. The numerical solutions of eq.(17) (solid lines) and their approximations due the eq.(19) (dashed lines). The inner, central, and outer curves are for $D = 0.2, 0.3, 0.4$, respectively.

to σ_8^2 and inversely proportional to the fraction of the scalar mode contributing to the large-scale temperature anisotropy variance, $S/(S+T)$. This simple argument makes eq.(16) independent of a particular way how it was obtained, just the proportionality coefficient should be taken properly.

Eq.(16) provides quite a general constraint on the fundamental inflation spectra (both S and T) in a wide set of dark matter models using only two basic measurement (the cluster abundance and large scale $\Delta T/T$). Actually, the DM information (a transfer function) is contained in the D -coefficient which can be calculated using the same equation (16) for a simple inflationary spectrum (e.g. power-law) preserving the given DM model. E.g. for CDM with $h = 0.5$ we have:

$$D \simeq \frac{0.6\sigma_8^2}{1 - 3.1\Omega_b}, \quad \Omega_b < 0.2. \quad (18)$$

The solution of eq.(17) is shown in the plane $x_2 - c$ for $D=0.2, 0.3, 0.4$ (see Fig. 2). For the whole range $0.1 < D < 0.5$, it can be analytically approximated with a precision better than 10% as follows:

$$\ln^2 \left(\frac{k_0}{k_{cr}} \right) \simeq E (c_0 - c) (c + c_1), \quad 2 < c \leq c_0. \quad (19)$$

Notice there exists not any solution of eq.(17) for high enough c ($c > c_0$). We have found the following best fit coefficients E , $k_0[h/\text{Mpc}]$ and $c_{0,1}$:

$$E \simeq 1, \quad \ln k_0 \simeq 49D^2 + 1.3, \\ c_0 \simeq 61D^2 + 6.2, \quad c_1 \simeq 44D^2 + 4.0.$$

The tensor-mode-contribution is approximated similarly:

$$\frac{T}{S} \simeq \frac{2.53 - 4.3D}{(\ln k_{cr} + 4.65)^{2/3}} + \frac{1}{3}. \quad (20)$$

Recall, k_{cr} is measured in the units h/Mpc .

4 DISCUSSION

We have presented a new inflationary model predicting a near scale-invariant spectrum of density perturbations and large amount of CGWs. Our model is consistent with COBE $\Delta T/T$ and cluster abundance data. The perturbation spectra depend on one free scale-parameter, k_{cr} , which can be found in further analysis by fitting other observational data. At the location of k_{cr} , the CDP spectrum transients smoothly from the red ($k < k_{cr}$) to the blue ($k > k_{cr}$) parts (see eq.(5)).

By adjusting k_{cr} with the galaxy cluster scale, we easily gain the boosts in power (in comparison with ΛCDM) on both scales, large (voids and superclusters) and small (quasars and Ly_α clouds). Say, for $k_{cr} = k_1$, $D = 0.4$ and $\Omega_b = 0.1$, we get $c \simeq 11$, $\sqrt{\lambda} \simeq 1.6 \cdot 10^{-6}$ and

$$T/S \simeq 0.7, \quad n_s \simeq 0.9 \quad (21)$$

at large scale ($\sim 1000h^{-1}\text{Mpc}$). The boost on Mpc-scale is about 8%,

$$\left(\frac{q_{10k_1}}{q_{k_1}} \right)^2 \simeq 1.08,$$

which is really a lot when compared with the red spectrum (21) extrapolated from large scales.

We conclude that the wing-like S-power-spectra similar to those in eq.(5), can provide a simple solution to the cosmological problem in a matter-dominated universe (cp. also a similar spectrum of CDPs in Semig, Müller 1996). Indeed, the spectrum power at the current dynamical scale is strongly suppressed by the present galaxy cluster abundance ($\sigma_8 \simeq 0.6$), whereas other ‘classical’ observations persistently require a large CDP power both on scales $\sim 100h^{-1}\text{Mpc}$ (the existence of large scale structures) and $\leq 1h^{-1}\text{Mpc}$ (QSOs, Ly_α forest, early galaxy formation).

More of that, today we seriously discuss a nearly flat shape of the dimensionless (linear) CDP-spectrum within the scale range encompassing clusters and superclusters,

$$\Delta_k^2 \sim k^{(0.9 \pm 0.2)}, \quad k \in (0.05, 0.2)h \text{ Mpc}^{-1}, \quad (22)$$

(with a break towards the HZ slope on higher scales) which stays in obvious disagreement with the ΛCDM predictions. The first arguments supporting eq.(22) came from the analysis of large-scale galaxy distribution (Guzzo et al. 1991) and the discovery of large quasar groups (Komberg, Lukash 1994, Komberg, Kravtsov, Lukash 1996). Recent measurements of the galaxy cluster power spectrum (Tadros, Efstathiou, Dalton 1998, Retzlaff et al. 1998) brought a higher statistical support for eq.(22) (see Fig.3).

A possible explanation of eq.(22) from a theoretical point of view can be a fundamental red power spectrum established on large scales; then the transition to the spectrum (22) at $\sim 100h^{-1}\text{Mpc}$ would be much easier understood with help of a traditional modification of the transfer function $T(k)$ (e.g. for mixed hot+cold dark matter, see Mikheeva et al. 2000). A boost of power of the high-multipole CMB anisotropy (at $l \sim 220$) looks self-consistent with the above argument on a slightly red S-spectrum at large scales. The redness may be not too high, remaining in the range (0.9, 1). Notice any introduction of a blue power spectrum at large scale appears extremely unfavorable in this connection

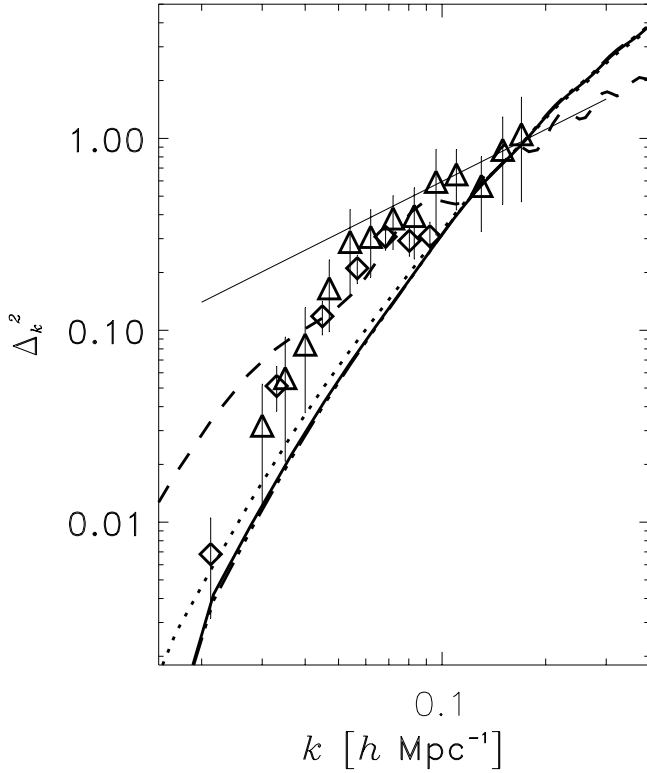


Figure 3. Power spectrum of Abell-ACO clusters (triangles) according to Retzlaff et al. (1998) and of APM clusters (diamonds) according to Tadros et al. (1998). The cluster power spectrum is reduced in amplitude to the galaxy level using a scale independent cluster bias factor $b_{cl} = 2$. We compare with 4 theoretical models, sCDM (dash-tree-dotted line), LCDM with $\Omega_\Lambda = 0.7$ (dashed), and the wing-like power spectra with parameters $c = 5, 11$ as dotted and solid lines, respectively. The $\Lambda\lambda$ models with $c = 5, 9, 11$ have mass variances $\sigma_8 = 0.4, 0.6, 0.7$, much lower than the unrealistic sCDM with $\sigma_8 = 1.1$. The $c = 11$ curve lies almost on top of sCDM, the (not shown) $c = 9$ curve lies between $c = 5$ and $c = 11$ curves. The solid straight line corresponds to the power law (22).

as it suppresses severely all the LSS effects at $\sim 100h^{-1}\text{Mpc}$ (provided the spectrum meets the σ_8 constraints).

The main problem for the LSS in matter-dominated models remains a low number of σ_8 : if $\sigma_8 \leq 0.6$, then the first acoustic peak in ΔT cannot be as high as $\geq 70 \mu K$ in any model with $\Omega_m = 1$ (see Fig.4, cp. a similar conclusion for MDM models in Mikheeva et al. 2000).

While the red power spectrum could help in solving the LSS problem, it is undesirable when continued to Mpc scale: here we need an enhanced power of CDPs to trigger the observed early structure formation. This is the way to see how a wing-like power spectrum can be of help.

Summarizing, our arguments are very simple:

(i) The wing-like S-power-spectra is a common feature of Λ -inflation;

(ii) An early galaxy formation period together with the developed structure at $(10 - 100)h^{-1}\text{Mpc}$ can be reconciled

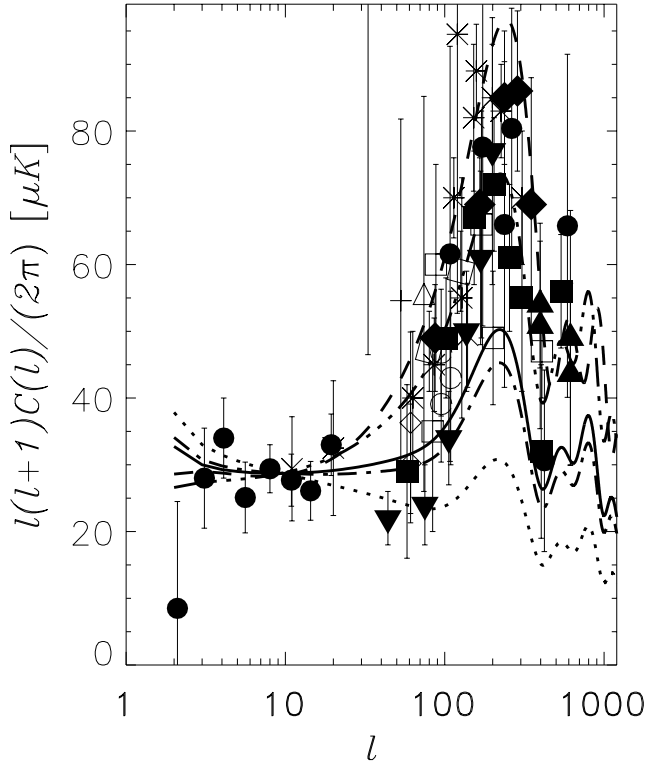


Figure 4. Multipole moments of the 5 theoretical models, sCDM (dash-three-dotted), Λ CDM (dashed), and the wing-like power spectra with $c = 5$ (dotted), $c = 9$ (dash-dotted), and $c = 11$ (solid line). The last curves have a tensor to scalar ratio T/S of 1.3, 0.9, and 0.7, respectively. For the calculation of the multipoles we used the Boltzmann code CMBFAST of Zaldarriaga, Seljak (1999). The observational data stem from the collection of experiments in Tegmark, Zaldarriaga (2000).

(for models with standard cold or mixed dark matter) in the considered wing-like primordial spectrum of CDPs.

Acknowledgments

The work was partly supported by German Scientific Foundation (DFG-436 RUS 113/357/0) and INTAS grant (97-1192).

REFERENCES

- Bennett C.L. et al., 1992, ApJ, 396, L7
- Bennett C.L. et al., 1996, ApJ, 464, L1
- Eke V.R., Coles S., Frenk C.S., 1996, MNRAS, 282, 263
- Garcia-Bellido J., Linde A., Wands D., 1996, Phys.Rev. D 54, 6040
- Gardner J.P., Katz N., Weinberg D.H., Hernquist L., 1997, ApJ, 486, 42
- Grishchuk L.P., 1974, Zh.Eksp.Teor.Fiz., 67, 825
- Guzzo L., Chincarini G., Iovino A., Giavanelli R., Haynes M.P., 1991, ApJ, 382, L5

- Hoyle F., Baugh C.M., Shanks T., Ratcliffe A., 1999, MNRAS, 309, 659
- Komberg B.V., Kravtsov A.V., Lukash V.N., 1996, A&A, 282, 713
- Komberg B.V., Lukash V.N., 1994, MNRAS, 269, 277
- Linde A., 1994, Phys.Rev. D, 49, 748
- Lucchin F., Matarrese S., 1985, Phys.Rev. D, 32, 1316
- Lukash V.N., 1980, Zh.Eksp.Teor.Fiz., 79, 1601
- Lukash V.N., 1996, in “Cosmology: The Physics of the Universe”, 213, eds. Robson B.A. et al., World Scientific; astro-ph/9910009
- Lukash V.N., Mikheeva E.V., 1996, Gravitation & Cosmology, 2, 247
- Lukash V.N., Mikheeva E.V., 1997, Electronic Proceeding of Particle Physics and the Early Universe Conference, University of Cambridge, <http://www.mrao.cam.ac.uk/ppeuc/astronomy/papers/lukash/lukash.html>
- Lukash V.N., Mikheeva E.V., 2000, Int.J.Mod.Phys.A, in press
- Mikheeva E.V., 1997, Large-scale anisotropy of CMB, Ph.D. Thesis, P.N.Lebedev Physical Institute, Moscow, in Russian
- Mikheeva E.V., Lukash V.N., Arhipova N.A., Malinovsky A.M., 2000, submitted to Russian Astron.J.
- Retzlaff J., Borgani S., Gottlöber S., Klypin A., Müller V., 1998, NewA 3, 631
- Rubakov V.A., Sazhin M.V., Veryaskin A.M., 1982, Phys.Lett. B, 115, 189
- Sachs R.K., Wolfe A.M., 1967, ApJ, 147, 73
- Semig L., Müller V., 1996, A&A, 308, 697
- Smoot G.J. et al., 1992, ApJ, 396, L1
- Starobinsky A.A., 1997, Zh.Eksp.Teor.Fiz, 30, 719
- Tadros H., Efstathiou G., Dalton C., 1998, MNRAS, 296, 995
- Tegmark M., Zaldarriaga M., 2000, ApJ submitted (astro-ph/0002091)
- Viana P.T., Liddle A.R., 1996, MNRAS, 281, 323
- White S.D.M., Efstathiou G., Frenk C.S., 1993, MNRAS, 262, 1023
- Zaldarriaga M., Seljak U., 1999, astro-ph/9911219

Appendix

Here, we recall the basic properties of the parametric amplification effect and derive the cosmological perturbation spectra originating in $\Lambda\lambda$ -inflation (Lukash, Mikheeva 2000).

The linear perturbations in the Friedmann geometry are irreducibly presented in terms of the uncoupled scalar and tensor parts:

$$ds^2 = (1 + h_{00}) dt^2 + 2ah_{0\alpha} dt dx^\alpha - a^2(\delta_{\alpha\beta} + h_{\alpha\beta}) dx^\alpha dx^\beta, \quad (A.1)$$

$$\frac{1}{2}h_{\alpha\beta} = A\delta_{\alpha\beta} + B_{,\alpha\beta} + G_{\alpha\beta}, \quad h_{0\alpha} = C_{,\alpha},$$

where $G_{\alpha\beta}$ is a trace-free divergence-less tensor field ($G_{\alpha\beta}^\alpha = G_{\alpha,\beta}^\beta = 0$), and potentials h_{00} , A , B , C are coupled to the perturbation of the scalar field φ (which is also the source of the Friedmann geometry). The vector mode is not included here for the absence of proper source.

The gauge-invariant canonical 4-scalar determining the physical scalar perturbations, $q = q(t, \vec{x})$, is uniquely fixed

by the appearance of the S-part of the perturbative Lagrangian similar to a massless field (Lukash 1980, 1996)^{||}:

$$\delta L \equiv L(q, G_{\alpha\beta}) = \gamma q_{,i} q^{,i} + \frac{1}{2} G_{\alpha\beta, \gamma} G^{\alpha\beta, \gamma}. \quad (A.2)$$

The relation of q to the original potentials have the following form:

$$\delta\varphi = \alpha(q + A), \quad a^2\dot{B} + C = \frac{\Phi + A}{H},$$

$$\frac{1}{2}h_{00} = \gamma q + \left(\frac{A}{H}\right)', \quad \Phi = \frac{H}{a} \int a\gamma q dt, \quad (A.3)$$

$$\frac{\delta\rho}{\rho + p} = \frac{\dot{q}}{H} - 3(q + A), \quad 4\pi G\delta\rho_c \equiv \gamma H\dot{q} = \frac{\Delta\Phi}{a^2},$$

where a , H , γ , $\rho = \dot{\varphi}^2/2 + V$, $\alpha = \dot{\varphi}/H = \pm\sqrt{2\gamma}$ are solutions of the Friedmann equations of the background cosmological model, i.e. they are pure time dependent, and $\Delta \equiv \partial^2/\partial\vec{x}^2$ is the spatial Laplacian. Any two potentials taken from the triple A , B , C are arbitrary functions of all coordinates specifying the gauge choice.

The equations of motions of the scalar and tensor fields propagating in the Friedmann Universe are two harmonic oscillators,

$$\ddot{q} + \left(3H + \frac{\dot{\gamma}}{\gamma}\right)\dot{q} - \frac{\Delta}{a^2}q = 0, \quad (A.4)$$

$$G_{\alpha\beta}'' + 3HG_{\alpha\beta}' - \frac{\Delta}{a^2}G_{\alpha\beta} = 0, \quad (A.5)$$

which reduce the problem of the generation of cosmological perturbations to the well established parametric amplification effect (particle creation in intensive gravitational fields).

The quantum-generated perturbations inflating outside the horizon become frozen in time and can be treated as classical random Gaussian fields with the variances

$$\langle q^2 \rangle = \int_0^{k_*} q_k^2 \frac{dk}{k}, \quad \langle G_{\alpha\beta} G^{\alpha\beta} \rangle = \int_0^{k_*} h_k^2 \frac{dk}{k}, \quad (A.6)$$

where $k_* = |\eta|^{-1}$ is the horizon-crossing time ($\eta \equiv \int dt/a$). The power spectra are given as usually in the limit $k|\eta| \ll 1$:

$$q_k = \frac{k^{\frac{3}{2}}|\nu_k|}{2\pi a\sqrt{\gamma}}, \quad h_k = \frac{k^{\frac{3}{2}}|\nu_k^\lambda|}{\pi a}, \quad (A.7)$$

by solving the respective Klein-Gordon equations for the functions $\nu_k^\lambda(\eta)$ (see (A.4), (A.5)),

$$\frac{d^2\nu_k^{(\lambda)}}{d\eta^2} + (k^2 - U^{(\lambda)})\nu_k^{(\lambda)} = 0, \quad (A.8)$$

with $U = d^2(a\sqrt{\gamma})/a\sqrt{\gamma}d\eta^2$, $U^\lambda = d^2a/d\eta^2$, and the vacuum initial conditions,

$$k|\eta| \gg 1: \quad \nu_k^{(\lambda)} = \frac{\exp(-ik\eta)}{\sqrt{2k}}. \quad (A.9)$$

^{||} For simplicity, we assume the total field Lagrangian density in the form $\frac{1}{2}\varphi_{,i}\varphi^{,i} - V(\varphi)$ and the Hilbert minimal action for gravity.

In particular, for slow-roll inflation (e.g. the $\Lambda\lambda$ -inflation) we have

$$\nu_k^{(\lambda)} \simeq \frac{\exp(-ik\eta)}{\sqrt{2k}} \left(1 - \frac{i}{k\eta} \right), \quad (\text{A.10})$$

and

$$q_k \simeq \frac{H}{2\pi\sqrt{2\gamma}}, \quad h_k \simeq \frac{H}{\pi\sqrt{2}}, \quad k = aH \simeq -1/\eta. \quad (\text{A.11})$$

The h_k spectrum takes into account both polarizations of CGWs.

In conclusion, we comment on eq.(13) recalling that $\Phi = 0.6q$ at the matter-dominated stage (see (A.3)), and

$$\frac{\Delta T}{T} = \frac{\Phi}{3} = \frac{q}{5} \quad (\text{A.12})$$

at large angular scale ($> 1^\circ$); both Newtonian and q potentials in (A.12) are taken at the last-scattering surface.